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A Variable Control Structure Controller for the Wing Rock Phenomenon

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ABSTRACT
This paper presents the design of a variable structure controller for the model of the wing rock phenomenon of a delta wing aircraft. It is considered to be a continue study of the last two researches for the same phenomena "Feedback linearization [15] and back stepping controller [14] ". A control technique is proposed to stabilize the aircraft phenomena. The solution presented in this paper give a guarantee of asymptotic convergence to zero of all variables of the system. MATLAB simulation used to show how the proposed control is working well for such phenomena of a delta wing aircraft. The model of the phenomena in this paper will consider the same model presented in the last two researches mentioned above.
Keywords: Wing Rock, Nonlinear Control of Wing Rock, Variable Structure Controller

I. INTRODUCTION

Wing-rock motion is a self-induced, limit-cycle rolling motion experienced by flight aircrafts with small aspect-ratio wings, or with long pointed forebodies at high angles of attack [1]. This phenomenon has been studied by many researchers, (see for example [1],[3],[4]) because of its importance in the stability of an aircraft during high angle of attack maneuvers. It was also reported in [6] that the oscillation that does not have a limit cycle can happened at an 80/65 degree double delta wing.

Wing rock is a nonlinear phenomenon experienced by aircraft in which oscillations and unstable sideslip behavior are experienced [9]. This instability may diminish flight effectiveness or even present a serious danger due to potential instability of the aircraft [1]. Wing rock has been extensively studied experimentally, resulting in mathematical models that describe the nonlinear rolling motion using simple differential equations as in [7],[8].

The wing rock model for a delta wing aircraft used in [1] is considered in this project. Wing rock is usually modeled as self-induced, pure rolling motion, which causes the rolling moment to be a nonlinear function of the roll angle ϕ and the roll-rate p. The coefficients of such nonlinear function are obtained by curve fitting with experimental data at specific values of angle of attack. In addition, yawing dynamic is added to the nonlinear function by considering the yawing rate r = - ($\partial\beta/\partial t$) and ignoring the nonlinear term involving β due to its small value compared with the other nonlinear terms.

II. MODEL OF THE WING ROCK PHENOMENON

Define the following variables:

ϕ : Bank angle "roll angle"
p =: Roll-rate (rad./s) ($p = \partial\phi/\partial t$).
δ : Aileron angle.
β : Sideslip angle.
$\frac{\partial\beta}{\partial t}$: Sideslip rate of change.

The differential equations of the system are obtained from experiments and data curve fitting, such that: The rolling moment is described by the following differential equation:

$$\frac{\partial p}{\partial t} = \mu p + f(\phi, p) + L_\beta \beta + L_\delta \delta \qquad \textbf{(2.1)}$$

where μ is the sting damping coefficient, L_β , L_δ are parameters.

The yawing moment is described by the following differential equation:

$$\frac{\partial^2 \beta}{\partial t^2} = -N_\beta \beta + N_r (\frac{\partial \beta}{\partial t}) - N_p p \qquad \textbf{(2.2)}$$

where N_β, N_r, N_p are parameters.

The differential equation for the first order aileron actuator is taken to be:

$$\partial \delta / \partial t = (u - \delta) / \tau \qquad \textbf{(2.3)}$$

where τ is the actuator time, and *u* is the controller.
The nonlinear self-induced rolling function $f(\phi, p)$ using five terms curve-fit [1] as follows:

$$f(\phi, p) = a_1 \phi + a_2 p + a_3 p^3 + a_4 \phi^2 p + a_5 \phi p^2 \qquad \textbf{(2.4)}$$

where coefficients a_1, a_2, a_3, a_4, a_5 are dependent on the angle of attack, taken to be 0.2 radian.

If the state variables are denoted by: $x = (\phi, p, \delta, \beta, \partial \beta / \partial t)^T$ then the state equations can be written as follows:

$$\begin{aligned}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= -\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^2(t) x_2(t) + b_1 x_2^3(t) + b_2 x_1(t) x_2^2(t) \\
&\quad + L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t) \\
\dot{x}_3(t) &= -k x_3(t) + k u \\
\dot{x}_4(t) &= x_5(t) \\
\dot{x}_5(t) &= -N_p x_2(t) - N_\beta x_4(t) - N_r x_5(t)
\end{aligned} \qquad \textbf{(2.5)}$$

The parametric values for the aerodynamics are:

Table 1: parametric values

a_1	-0.05686
a_2	0.03254
a_3	0.07334
a_4	-0.3597
a_5	1.4681
μ_1	0.354* a_2 -0.001
μ_2	0.354* a_3
b_1	0.354* a_4
b_2	0.354* a_5
ω^2	0.354* a_1
L_δ	1
L_β	-0.02822
L_r	0.1517
k	1/0.0495
N_p	-0.0629
N_β	1.3214
N_r	-0.2491

As an oscillating system, the dynamics of wing rock phenomenon with no control will be unstable and oscillating with limit cycle motion. The unstable behavior on the aircraft's wings appears with undesirable yawing motion in the flight, which might cause serious damage. To see such instable oscillating dynamics of the phenomenon, we can plot the states with no control (u =0). Figure 1 – Figure 5 show the plots of $\phi, p, \delta, \beta, \frac{\partial \beta}{\partial t}$ respectively.

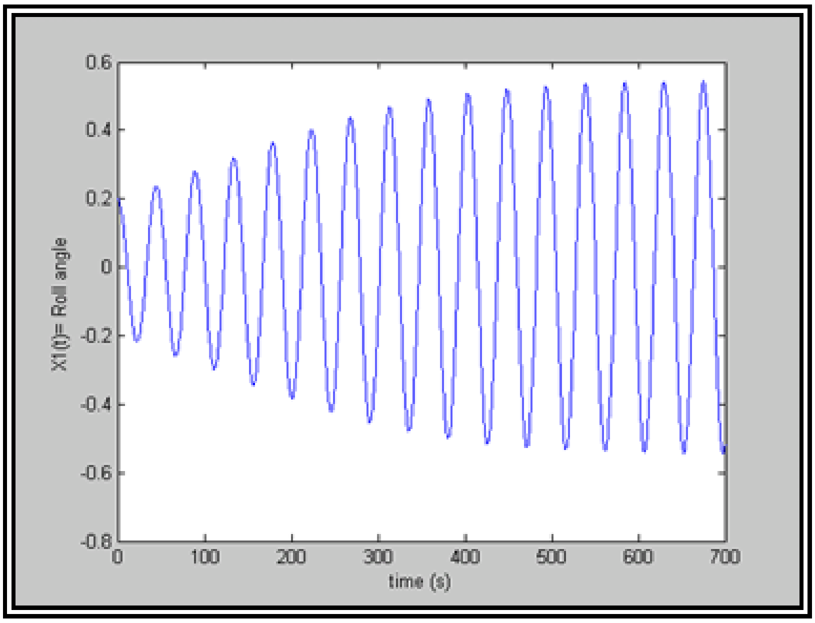

Figure 1: $\phi =$ roll angle (rad.)

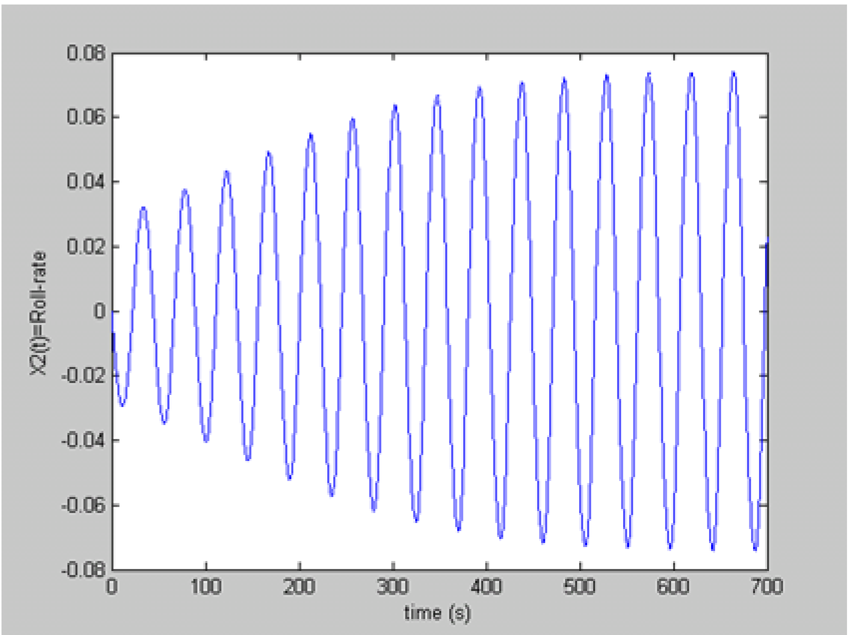

Figure 2 p = roll-rate (rad./s)

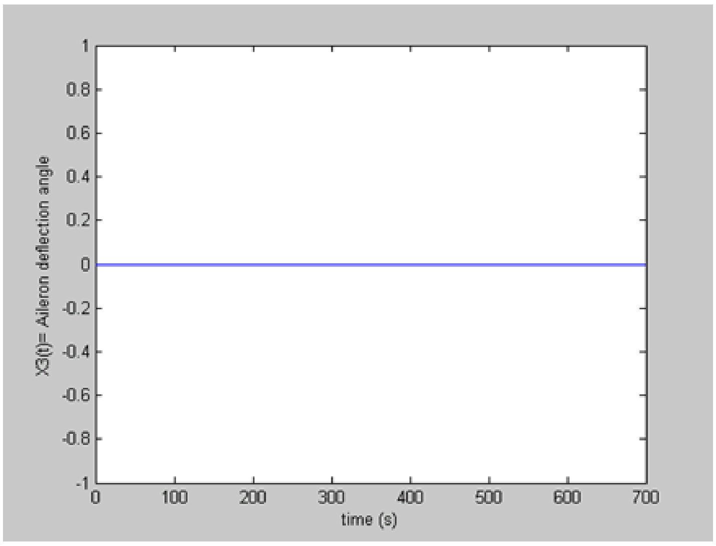

Figure 3 δ = aileron angle (rad.)

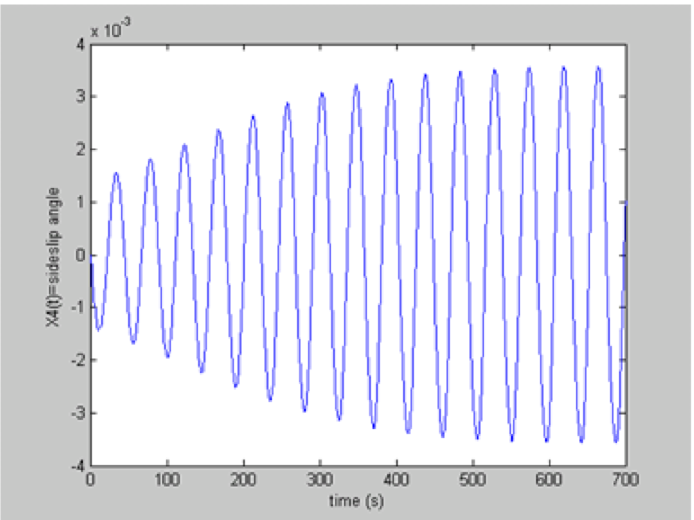

Figure 4 β sideslip angle (rad.)

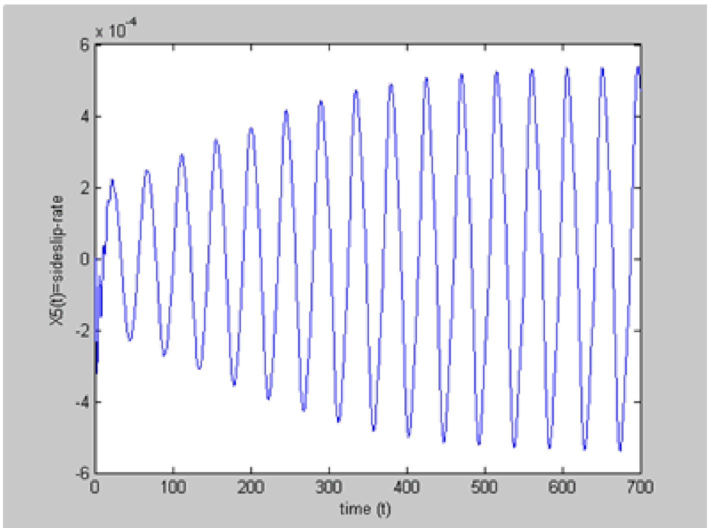

Figure 5 $\partial\beta/\partial t$ sideslip rate of change (rad/s)

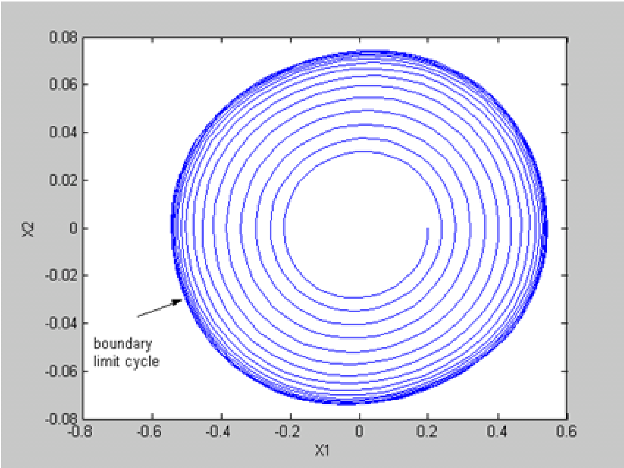

Figure 6 Roll rate vs. Roll angle

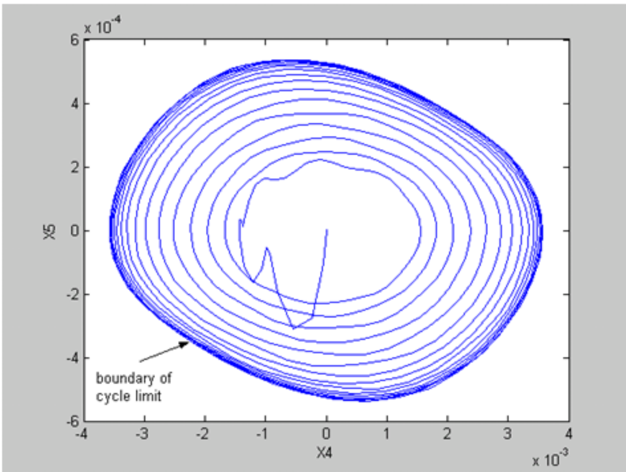

Figure 7 sideslip Rate vs. Sideslip angle

Transformation Function T (x)

The dynamic of the wing rock phenomenon is highly nonlinear. Therefore, a nonlinear transformation $z = T(x)$ [2] will be used to transfer the dynamic model of the system into a form that will simplify the design of nonlinear control schemes.

The transformation $z = T(x)$ is defined such that:

$$z_1 = N_p x_1 + N_r x_4 + x_5$$

$$z_2 = -N_\beta x_4$$

$$z_3 = -N_\beta x_5 \qquad \textbf{(2.6)}$$

$$z_4 = N_\beta (N_p x_2 + N_\beta x_4 + N_r x_5)$$

$$z_5 = N_\beta N_p (-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^{\,2}(t) x_2(t) + b_1 x_2^{\,3}(t) + b_2 x_1(t) x_2^{\,2}(t)$$

$$+ L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)$$

The inverse transformation $x = T^{-1}(z)$ exists and it is as follows.

$$x_1 = \frac{1}{N_p}(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)$$

$$x_2 = \frac{1}{N_\beta N_p}(z_4 + N_\beta z_2 + N_r z_3)$$

$$x_3 = \frac{1}{L_\delta}[z_5 + (\omega^2 N_\beta - \frac{b_2}{N_\beta N_p^2}(z_4 + N_\beta z_2 + N_r z_3)^2)(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)]$$
$$-\frac{1}{L_\delta}[(\mu_1 + \frac{\mu_2}{N_p^2}(z_1 + \frac{N_r}{N_\beta} z_2 + \frac{1}{N_\beta} z_3)^2 + \frac{b_1}{N_\beta^2 N_p^2}(z_4 + N_\beta z_2 + N_r z_3)^2)(z_4 + N_\beta z_2 + N_r z_3)]$$
$$+\frac{1}{L_\delta}[N_p L_\beta z_2 - N_p L_r z_3 + N_\beta z_3 + N_r z_4]$$

$$x_4 = -\frac{1}{N_\beta} z_2$$

(2.7)

$$x_5 = -\frac{1}{N_\beta} z_3$$

Hence, the dynamic model of the wing rock phenomenon can be written as,

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = z_5$$

$$\dot{z}_5 = q(x) + g(x)u$$

(2.8)

where:

$$q(x) = \left[N_\beta N_p(-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1^{\,2}(t) x_2(t) + b_1 x_2^{\,3}(t) + b_2 x_1(t) x_2^{\,2}(t) + + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r(N_p x_2 + N_\beta x_4 + N_r x_5)\right] - L_\delta k\, x_3(t)$$

$$g(x) = k N_\beta N_p L_\delta$$

III. VARIABLE STRUCTURE CONTROLLER FOR THE WING ROCK PHENOMENON

Recall from the previous chapter that the wing rock phenomenon can be described using the following set of differential equations:

$$\dot{x}_1(t) = x_2(t)$$
$$\dot{x}_2(t) = -\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t)$$
$$+L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)$$
$$\dot{x}_3(t) = -k x_3(t) + ku$$
$$\dot{x}_4(t) = x_5(t)$$
$$\dot{x}_5(t) = -N_p x_2(t) - N_\beta x_4(t) - N_r x_5(t) \qquad \textbf{(3.1)}$$

Using the transformation defined in chapter 2, the above equations can be written as:

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = z_5$$

$$\dot{z}_5 = q(x) + g(x)u$$
(3.2)

With,

$$q(x) = [N_\beta N_p(-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t) + +L_\beta x_4(t) - L_r x_5(t))$$
$$+N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)] - L_\delta k\, x_3(t)$$

$$g(x) = k N_\beta N_p L_\delta$$

Let the scalars $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ be chosen such that the polynomial $P_2(s) = s^4 - \alpha_1 s^3 - \alpha_2 s^2 - \alpha_3 s - \alpha_4$ is a Hurwitz polynomial (i.e. , the roots of $P_2(s) = 0$ are located in the left half plane). Also, let W be a positive scalar.

Define the following sliding surface for the above system:

$$\sigma = z_5 - \alpha_1 z_1 - \alpha_2 z_2 - \alpha_3 z_3 - \alpha_4 z_4 \qquad \textbf{(3.3)}$$

To guarantee the reachability to the surface $\sigma = 0$, we impose the following dynamics on the sliding surface,

$$\dot{\sigma} = -W\ sign(\sigma) \qquad \textbf{(3.4)}$$

It can be easily checked that the reacheability condition is satisfied by using the dynamics given in (5.4) .
Let the lyapunov function *V* be such that:

$$V = \frac{1}{2}\sigma^2 \quad \textbf{(3.5)}$$

Taking the derivative of V with respect to time, it follows that:

$$\dot{V} = \sigma\dot{\sigma} = -W\sigma sign(\sigma) = -W\frac{\sigma^2}{|\sigma|} \quad < \quad 0. \quad \textbf{(3.6)}$$

On the sliding surface, the reduced order dynamics of the system is as follows:

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = z_4$$

$$\dot{z}_4 = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4 \quad \textbf{(3.7)}$$

Define the vector z such that $z_p = [z_1 \quad z_2 \quad z_3 \quad z_4]^T$.

The closed loop system given by (5.7) can be written in compact form as:

$$\dot{z}_p = A_{zp} z_p \quad \textbf{(3.8)}$$

where the matrix A_{zp} is such that:

$$\therefore A_{zp} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{bmatrix}$$

The solution of the differential equation given by (5.8) is $z_p(t) = \exp(A_{zp}t)z_p(0)$. Since the matrix A_{zp} is a stable matrix, the vector $z_p(t)$ will converge to zero asymptotically as $t \to \infty$. Hence z_1, z_2, z_3, z_4 will converge to zero asymptotically as $t \to \infty$. Moreover, on the switching surface $\sigma = 0$ which implies that $z_5 - \alpha_1 z_1 - \alpha_2 z_2 - \alpha_3 z_3 - \alpha_4 z_4$ =0. Thus it can be concluded that $z_5 = \alpha_1 z_1 + \alpha_2 z_2 + \alpha_3 z_3 + \alpha_4 z_4$ will converge to zero asymptotically as $t \to \infty$.

The dynamics in (5.4) can be written as: $\dot{\sigma} = -W\ sign(\sigma)$.
or,

$$\dot{z}_5 - \alpha_1\dot{z}_1 - \alpha_2\dot{z}_2 - \alpha_3\dot{z}_3 - \alpha_4\dot{z}_4 = -sign(\sigma) \Rightarrow$$

$$q(x) + g(x)u - \alpha_1 z_2 - \alpha_2 z_3 - \alpha_3 z_4 - \alpha_4 z_5 = -Wsign(\sigma) \quad \textbf{(3.9)}$$

Solving the above equation for u leads to :

$$u = \frac{1}{g(x)}[-q(x) + \alpha_1 z_2 + \alpha_2 z_3 + \alpha_3 z_4 + \alpha_4 z_5 - Wsign(\sigma)] \quad \textbf{(3.10)}$$

Therefore, the previous development allows us to state the following proposition:

The variable structure controller

$$u = \frac{1}{g(x)}[-q(x) + \alpha_1 z_2 + \alpha_2 z_3 + \alpha_3 z_4 + \alpha_4 z_5 - Wsign(\sigma)] \quad \textbf{(3.11)}$$

guarantees the asyptotic convergence of z_1, z_2, z_3, z_4, z_5 to zero as $t \to \infty$.

Remark
The variable structure controller in (3.11) can be written in the original coordinates by using the transformation:

$$z_1 = N_p x_1 + N_r x_4 + x_5$$

$$z_2 = -N_\beta x_4$$

$$z_3 = -N_\beta x_5$$

(3.12)

$$z_4 = N_\beta (N_p x_2 + N_\beta x_4 + N_r x_5)$$

$$z_5 = N_\beta N_p (-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t)$$

$$+L_\delta x_3(t) + L_\beta x_4(t) - L_r x_5(t)) + N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)$$

And

$$q(x) = [N_\beta N_p (-\omega^2 x_1(t) + \mu_1 x_2(t) + \mu_2 x_1{}^2(t) x_2(t) + b_1 x_2{}^3(t) + b_2 x_1(t) x_2{}^2(t) + + L_\beta x_4(t) - L_r x_5(t))$$
$$+N_\beta^2 x_5 - N_\beta N_r (N_p x_2 + N_\beta x_4 + N_r x_5)] - L_\delta k\, x_3(t)$$

(3.1

$$g(x) = kN_\beta N_p L_\delta$$

3)

3.1 Simulation results
The poles of $P_2(s) = 0$ are chosen to be -1, -2, -3, -4, then

$$P_2(s) = s^4 - \alpha_1 s^3 - \alpha_2 s^2 - \alpha_3 s^1 - \alpha_4 = (s+1)(s+2)(s+3)(s+4) \quad \Rightarrow$$

$$s^4 - \alpha_1 s^3 - \alpha_2 s^2 - \alpha_3 s^1 - \alpha_4 = s^4 + 10s^3 + 35s^2 + 50s^1 + 24 \quad \Rightarrow$$

$$\alpha_1 = -10, \quad \alpha_2 = -35, \quad \alpha_3 = -50, \quad \alpha_4 = -24$$

The simulations is done using MAT LAB and the results plotted for the states for initial values ={0.2 0 0 0 0} and W=0.01. Figure 18 – Figure 22 show the plots of $\phi,\ p,\ \delta,\ \beta,\ \frac{\partial \beta}{\partial t}$ respectively.

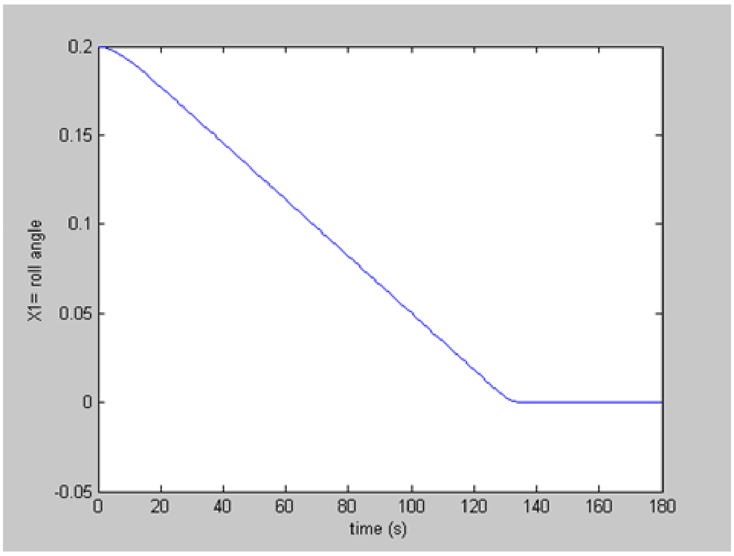

Figure 18 Roll angle (rad.)

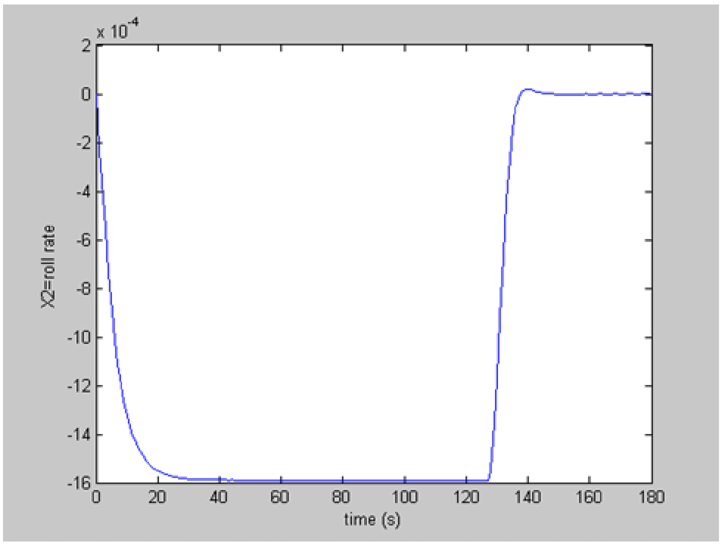

Figure 19 Roll-rate (rad/sec)

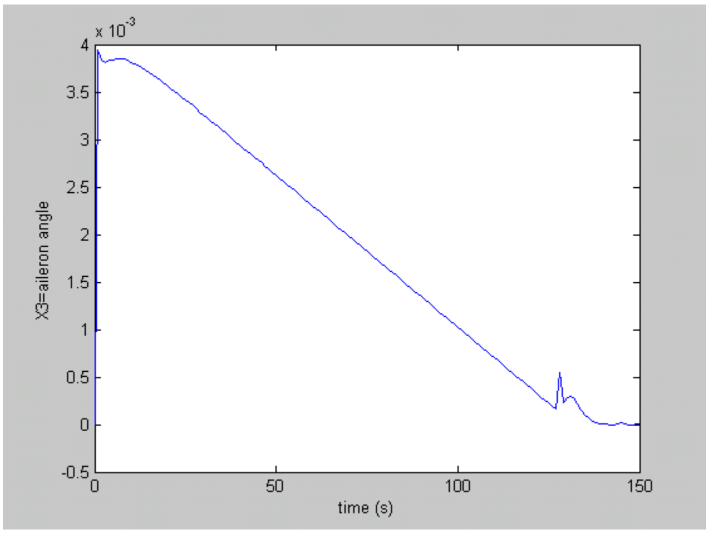

Figure 20 Aileron deflection angle (rad.)

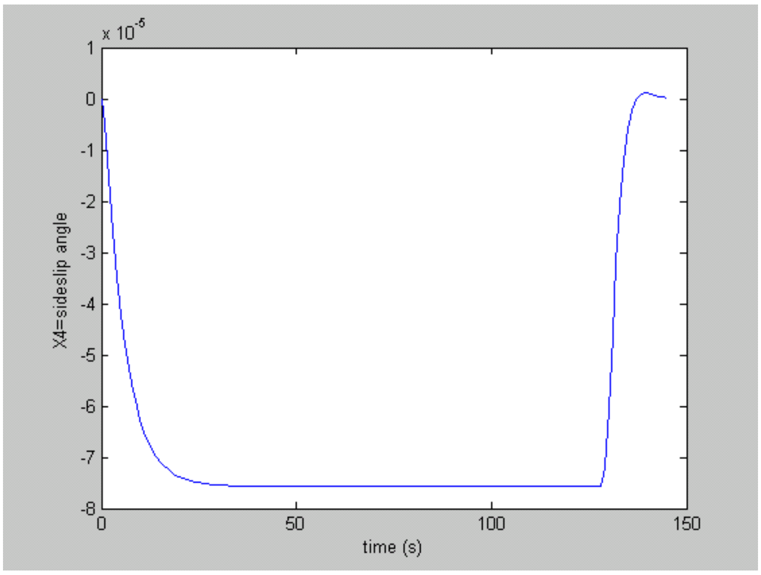

Figure 21 sideslip angle (rad.)

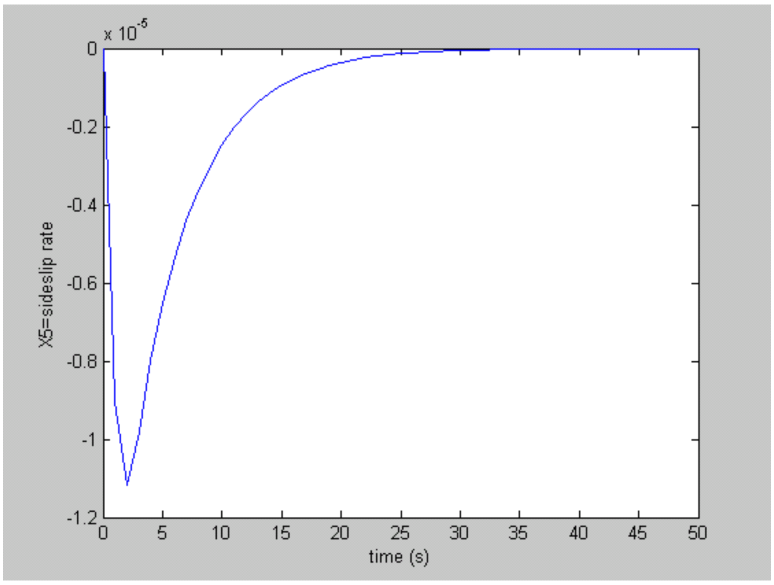

Figure 22 sideslip rate (rad/sec)

IV. CONCLUSION

The results insure that the system considered for such phenomenon converge to zero asymptotically with suggested control solution. And the proposed variable structure controller stabilizes the system well.

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